2018 EWM-EMS Summer School: Nonlocal interactions in Partial Differential Equations and Geometry

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Book of Abstracts

and List of Participants

Contents

An introduction to the fractional Yamabe problem

María del Mar González

Abstract

The aim of this course is to present one very recent topic in differential geometry: the fractional Yamabe problem, both from the analytic and geometric points of view, but especially towards the PDE community interested in nonlocal problems.

The basic tool in conformal geometry is to understand the geometry of a space by studying the transformations on such space. The simplest differential operator with conformal properties is the conformal Laplacian $L_h := -\Delta_h + \frac{n-2}{4(n-1)} R_h$, and its associated curvature (which is precisely the scalar curvature R_h). It gives rise to interesting semilinear equations with reaction term of power type, such as the constant scalar curvature (or Yamabe) equation

$$-\Delta_h u + \frac{n-2}{4(n-1)} R_h = \frac{n-2}{4(n-1)} c u^{\frac{n+2}{n-2}}.$$

The Yamabe problem has been one of the multiple examples of the interaction between analysis and geometry.

A higher order example of a conformally covariant operator is the Paneitz operator, which is defined as the bi-Laplacian $(-\Delta_h)^2$ plus lower order curvature terms. Being M a Riemannian manifold with a Riemannian metric h, these operators belong to a general framework in which on the manifold (M, h) there exists a meromorphic family of conformally covariant pseudodifferential operators of fractional order

$$P_s^h = (-\Delta_h)^s + \dots \quad \text{for any } s \in (0, n/2).$$

P_h^s will be called the conformal fractional Laplacian. The main goal of this course is to describe and to give some examples, applications and open problems for this non-local object.

We will explain the construction of the conformal fractional Laplacian from a purely analytical point of view. Caffarelli and Silvestre

gave a construction for the standard fractional Laplacian as a Dirichlet-to-Neumann operator of a uniformly degenerate elliptic boundary value problem. In the manifold case, Chang and the lecturer related the original definition of the conformal fractional Laplacian coming from scattering theory (in four-dimensional gravitational Physics) to a Dirichlet-to-Neumann operator for a related elliptic extension problem, thus allowing for an analytic treatment of Yamabe-type problems in the non-local setting.

The fractional Yamabe problem, poses the question of finding a constant fractional curvature metric in a given conformal class. From the geometric point of view, the fractional Yamabe problem is a generalization of Escobar's classical problem on the construction of a constant mean curvature metric on the boundary of a given manifold. In the simplest case, the resulting (non-local) PDE is

$$(-\Delta)^s u = c u^{\frac{n+2s}{n-2s}} \qquad \text{in } \mathbb{R}^n, \;\; u > 0.$$

The underlying idea is to pass to the extension, looking for a solution of a (possibly degenerate) elliptic equation with a nonlinear boundary reaction term, which can be handled through a variational argument where the main difficulty is the lack of compactness. As in the usual Yamabe problem, the proof is divided into several cases; some of them still remain open.

The next goal of the course would be to consider the singular Yamabe problem, this is, to find a metric with constant fractional curvature that is singular at a given set. Restrictions on the curvature will give geometric and topological restrictions for this construction. In particular, there is a nice construction of Delaunay-type solutions in the case of an isolated singularity. This topic involves the use of ODE type methods for non-local equations, which seems to be new.

Geometric aspects of phase separation

Susanna Terracini

Abstract

Several physical phenomena can be described by a certain number of densities (of mass, population, probability, ...) distributed in a domain and subject to laws of diffusion, reaction, and *competitive interaction.* Whenever the competitive interaction is the prevailing phenomenon, the several densities can not coexist and tend to segregate, hence determining a partition of the domain *(Gause's experimental principle of competitive exclusion (1932)).* As a model problem, we consider the system of stationary equations

$$-\Delta u_i = f_i(u_i) - \beta u_i \sum_{j\neq i} g_{ij}(u_j) \qquad u_i > 0\ .$$

The cases $g_{ij}(s) = \beta_{ij}s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij}s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

In this series of lectures, we will undertake the analysis of qualitative properties of solutions to systems of semilinear elliptic equations, whenever the parameter β, accounting for the competitive interactions, diverges to infinity. At the limit, when the minimal interspecific competition rate $\beta = \min_{ij}\beta_{ij}$ diverges to infinity, we find a vector $U = (u_1, \cdots, u_h)$ of functions with mutually disjoint supports: *the segregated states*: $u_i \cdot u_j \equiv 0$, for $i \neq j$, satisfying

$$-\Delta u_i = f_i(x, u_i) \qquad\qquad \text{whenever } u_i \neq 0\ ,\quad i = 1, \dots, h.$$

We will consider the following aspects:
(a) Spectral problems: optimal partitions w.r.t. eigenvalues in connection with monotonicity formulæ.
(b) Entire solutions of the competitive elliptic system:

$$-\Delta u_i = -\sum_{j\neq i} u_i u_j^2 \ \text{ in } \mathbb{R}^N, \qquad u_i > 0 \ \text{ in } \mathbb{R}^N \qquad i = 1, \dots, k.$$

(c) Competition-diffusion problems with fractional Laplacians.
(d) Competition-diffusion problems with nonlocal interactions.
(e) Spiralling solutions in the non symmetrical case.

Qualitative properties of solutions to some nonlocal problems in nonbounded domains

Begoña Barrios Barrera

Departamento de Análisis Matemático
University of La Laguna (ULL). Tenerife. Spain

bbarrios@ull.es

Along this talk we will be interested in the study of qualitative properties of classical solutions of semilinear nonlocal problems that involve the fractional Laplacian operator $(-\Delta)^s$, $0 < s < 1$, that are posed in the half-space $\mathbb{R}^N_+ = \{x = (x', x_N) \in \mathbb{R}^N : \ x_N > 0\}$, $N \geq 2$ or the whole real line $\mathbb{R}$.
Concerning with the problem in the half space $(-\Delta)^s u = f(u)$, $\mathbb{R}^N_+$, $u = 0, \mathbb{R}^N \setminus \mathbb{R}^N_+$, we will show that

- If $f \in C^1$, with no additional restriction on the function f, bounded, nonnegative, nontrivial classical solutions are indeed positive in $\mathbb{R}^N_+$ and verify $\frac{\partial u}{\partial x_N} > 0 \quad$ in $\mathbb{R}^N_+$. This is in contrast with previously known results for the local case $s = 1$ (see for instance [5, 6]).

- If $f \in Lip_{loc}$, using a complete characterization of one-dimensional solutions, that if u is a bounded solution with $\rho := \sup_{\mathbb{R}^N} u$ verifying $f(\rho) = 0$, then, as in the classical case ([4]), u is necessarily one-dimensional.

In the case of the study of $(-\Delta)^s u = f(u)$ in $\mathbb{R}$ we establish, uder natural hypothesis on f, the existence of periodic solutions introducing a suitable framework which allows to reduce the search for such solutions to the resolution of a boundary value problem in a suitable Hilbert space, thereby making it possible to reach for the usual tools of nonlinear analysis, like bifurcation theory or variational methods.

The results presented in this talk can be found in [1, 2, 3].

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Unique continuation and classification of blow-up profiles for fractional elliptic equations

Veronica Felli

In this talk I will describe the use of Almgren type monotonicity formulas to establish unique continuation principles for elliptic operators. I will show how a combination of monotonicity with blow-up analysis allows obtaining not only unique continuation but also precise asymptotics of solutions, by extracting such precious information from the behavior of the quotient associated with the Lagrangian energy. Finally I will discuss unique continuation properties for fractional elliptic equations and give a precise description of possible blow-up profiles in terms of a Neumann eigenvalue problem on the sphere.

Singularity formation in critical parabolic equations

Monica Musso

In this talk I will discuss some recent constructions of blow-up solutions for a Fujita type problem for power related to the critical Sobolev exponent. Both finite type blow-up (of type II) and infinite time blow-up are considered. This research program is in collaboration with C. Cortazar, M. del Pino and J. Wei.

Recent developments in the thin obstacle problem

Mariana Smit Vega Garcia

The study of the classical obstacle problem began in the 60's with the pioneering works of G. Stampacchia, H. Lewy and J. L. Lions. During the past five decades it has led to beautiful and deep developments in calculus of variations and geometric partial differential equations. One of its crowning achievements has been the development, due to L. Caffarelli, of the theory of free boundaries. Nowadays the obstacle problem continues to offer many challenges and its study is as active as ever. In particular, over the past years there has been some interesting progress the thin obstacle problem, also called Signorini problem. In this talk I will overview the thin obstacle problem for a divergence form elliptic operator, and describe a few methods used to tackle two fundamental questions: what is the optimal regularity of the solution, and what can be said about the free boundary, in particular the regular and singular sets. The proofs are based on Almgren, Weiss and Monneau type monotonicity formulas. This is joint work with Nicola Garofalo and Arshak Petrosyan.

An extension problem and Hardy type inequalities for fractional powers of the Grushin operator

Sundaram Thangavelu

In this talk we report some recent results on the extension problem associated to the Grushin operator $G = -\Delta - |x|^2\partial_w^2$ on $\mathbb{R}^{n+1}$. We use the solutions to prove trace Hardy and Hardy inequalities for conformally invariant fractional powers G_s of G. We also prove Hardy-Littlewood-Sobolev inequalities for G_s which are optimal when n is even.

On the nodal set of solutions to a nonlocal Heat Equation

Alessandro Audrito*

Università degli Studi di Torino and Politecnico di Torino

In this talk, I will present some recent results obtained in collaboration with Susanna Terracini concerning the nodal properties of solutions to a nonlocal parabolic equation.
We prove bounds on the Hausdorff dimension of the nodal set and a *stratification* property of its *singular* part by properly combining a Poon type monotonicity formula, a blow-up procedure, and some classical results as the Federer's reduction principle and the Whitney's extension. The analysis of the singular set presents the most significative differences w.r.t. the classical case. Finally, we describe the asymptotic behaviour of solutions near their nodal points in terms of linear combinations of a special class of parabolically homogeneous polynomials.

*Also affiliated with Universidad Autónoma de Madrid,Spain.

On the asymptotic behaviour of nonlocal perimeters

Judith Berendsen

We study a class of integral functionals known as nonlocal perimeters, which, intuitively, express a weighted interaction between a set and its complement. The weight is provided by a positive kernel K, which might be singular. The existence of minimisers for the corresponding Plateau?s problem is shown. Also, when K is radial and strictly decreasing, we present the result that halfspaces are minimisers if we prescribe "flat" boundary conditions. Furthermore, a Γ-convergence result is discussed. We study the limiting behaviour of the nonlocal perimeters associated with certain rescalings of a given kernel that has faster-than-L^1 decay at infinity and we show that the Γ-limit is the classical perimeter up to a multiplicative constant that can be computed explicitly.

On unbounded solutions of ergodic problems for non-local Hamilton-Jacobi equations

Cristina Brändle

We study an ergodic problem associated to a non-local Hamilton-Jacobi equation defined on the whole space $\lambda - \mathcal{L}[u](x) + |Du(x)|^m = f(x)$ and determine whether (unbounded) solutions exist or not. We prove that there is a threshold growth of the function f, that separates existence and non-existence of solutions, a phenomenon that does not appear in the local version of the problem. Moreover, we show that there exists a critical ergodic constant, λ_*, such that the ergodic problem has solutions for $\lambda \leq \lambda_*$ and such that the only solution bounded from below, which is unique up to an additive constant, is the one associated to λ_*.

Some recent results in the study of fractional mean curvature flow

Eleonora Cinti

We study a geometric flow driven by the fractional mean curvature (FMC). The notion of fractional mean curvature arises naturally when performing the first variation of the fractional perimeter functional (introduced by Caffarelli, Roquejoffre, and Savin). More precisely, we show the existence of surfaces which develope neckpinch singularities, in any dimension $n \geq 2$. Interestingly, in dimension $n = 2$ our result gives a counterexample to Greyson Theorem for the classical mean curvature flow. The result has been obtained in collaboration with C. Sinestrari and E. Valdinoci.

Growing sandpiles on networks

Lucilla Corrias

I'll consider a system of differential equations of Monge-Kantorovich type, which describes the equilibrium configurations of granular material poured by a constant source on a network. Relying on the definition of viscosity solution for Hamilton-Jacobi equations on networks introduced in P.-L. Lions and S. P. E., Viscosity solutions for junctions: well posedness and stability, Rend. 902 Lincei Mat. Appl., 27 (2016), pp. 535–545., existence and uniqueness of the solution of the system is proved. Some numerical test will be also shown.

This is a joint work with F. Camilli (La Sapienza Università di Roma) and S. Cacace (Università degli Studi di Roma Tre).

“Improved Adams-type inequalities and their extremals in dimension $2m$.”

Azahara DelaTorre

This talk is based on the proof of the existence of extremal functions for the Adams-Moser-Trudinger inequality on the Sobolev space $H^m(\Omega)$, where Ω is any bounded, smooth, open subset of $\mathbb{R}^{2m}$, $m \geq 1$. Moreover, we extend this result to improved versions of Adams' inequality of Adimurthi-Druet type. Our strategy is based on blow-up analysis for sequences of subcritical extremals and introduces several new techniques and constructions. The most important one is a new procedure for obtaining capacity-type estimates on annular regions.

This is a joint work with Gabriele Mancini.

The Talbot effect and Riemann's non-differentiable functions

Daniel Eceizabarrena

The **Talbot effect** is a microscopic optic phenomenon generated by the diffraction of light. Discovered in 1836 by Fox Talbot, it describes the pattern that light forms after going through a periodic grating, and more particularly, the repetition of the image of the grating itself. **Riemann's non-differentiable function** was proposed by Bernhard Riemann in 1872 as an (at the time) rare example of the possibility that continuous functions need not have a derivative. These apparently unrelated phenomena turn out to go hand in hand thanks to the chance of modelling the Talbot effect by means of the Schrödinger equation. In the talk, I will explain this relation with some detail

On nonlocal (and local) equations of porous medium type

Jørgen Endal

We study uniqueness, existence, and properties of bounded distributional solutions of the initial value problem for nonlocal (and local) equations of porous medium type. Here the nonlocal operator can be any symmetric degenerate elliptic operator including the fractional Laplacian and some of its own numerical discretizations (!). The nonlinearity is only assumed to be continuous and nondecreasing. The class of equations include (generalized) porous medium equations, fast diffusion equations, and Stefan problems. This is a joint work with Félix del Teso and Espen R. Jakobsen at NTNU.

The Schrödinger equation on star-graphs under general coupling conditions

Andreea Grecu

The aim of this talk is to present dispersive and Strichartz estimates for the Schrödinger time evolution propagator e^{-itH} on a star-shaped metric graph. The linear operator, H, taken into consideration is the self-adjoint extension of the Laplacian, subject to the wide class of coupling conditions introduced in the literature by Kostrykin and Schrader. The study relies on an explicit spectral representation of the solution in terms of the resolvent kernel which is further analyzed using results from oscillatory integrals. As an application, we obtain the global well-posedness for a class of semilinear Schrödinger equations. This is based on a joint work with Liviu Ignat.

A Lipschitz metric for the Hunter–Saxton equation

Katrin Grunert

Solutions of the Hunter–Saxton equation might experience wave breaking in finite time, i.e. their spatial derivative might become unbounded from below, while the solution itself remains bounded. Thus solutions in general do not exist globally, but only locally in time. In addition, energy concentrates on sets of measure zero when wave breaking occurs. The prolongation of solutions beyond wave breaking is therefore non-unique. We show how the stability of conservative solutions, i.e., solutions where the energy is not manipulated at breaking time, can be analyzed by constructing a Lipschitz metric, which is based on Wasserstein metrics, in the case of the Hunter–Saxton equation.

This is joint work with J.A. Carrillo and H. Holden.

A framework for non-local, non-linear diffusion

Miłosz Krupski

Diffusion is a ubiquitous notion in the theory of PDEs. The most obvious case is the heat equation and it has many derivations, including both non-local and non-linear examples (fractional Laplacian, fractional p-Laplacian, porous medium). We will discuss how to make your own diffusion operator from scratch and why it will have (some of) the properties you would like it to have. Joint work with G. Karch (Wrocław) and M. Kassmann (Bielefeld).

On fractional Poincaré inequalities

Javier Martínez Perales

In this talk we will discuss geometric conditions for a bounded domain to support fractional versions of Poincaré and Poincaré-Sobolev inequalities. Moreover, we will talk about *improved Poincaré inequalities* (following the nomenclature by Irene Drelichman and Ricardo G. Durán in [1]). What is obtained at the right hand side of the inequality is the seminorm defining the real interpolation space between the corresponding L^p and $W^{1,p}$ spaces on the domain, as it can be found in [2]. The topic of the talk is related with an ongoing work with Eugenia Cejas (Universidad Nacional de La Plata, Argentina) and Irene Drelichman (Universidad de Buenos Aires, Argentina) and it is based on the work by Ritva Hurri-Syrjänen and Antti V. Vähäkangas in [3].

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A nondegenerate solution for the Yamabe problem with maximal rank

María Medina de la Torre *

In this talk we will construct a sequence of nondegenerate (in the sense of Duyckaerts-Kenig-Merle ([2]) nodal nonradial solutions to the critical Yamabe problem

$$-\Delta u = \frac{n(n-2)}{4}|u|^{\frac{4}{n-2}}u, \qquad u \in \mathcal{D}^{1,2}(\mathbb{R}^n),$$

which, for $n = 4$, provides the first example in the literature of a solution with *maximal rank*.

This is a joint work with M. Musso and J. Wei that can be found at arxiv.org/pdf/1712.00326.pdf.

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*Departamento de Matemática, Pontificia Universidad Católica de Chile, e-mail: mamedinad@mat.uc.cl

Sharp Convergence Rate of Eigenvalues in a Domain with a Shrinking Tube

Roberto Ognibene

Consider a bounded domain in $\mathbb{R}^N$ with a bounded, thin tube attached to it. Basing ourselves on an Almgren-type monotonicity formula and on the Courant-Fischer Min-Max characterization, we provide the sharp asymptotic behaviour of a fixed simple eigenvalue of the Dirichlet-Laplacian, when the section of the tube is vanishing.

Quantitative a Priori Estimates for Fast Diffusion Equations with Caffarelli-Kohn-Nirenberg weights. Harnack inequalities and Hölder continuity

N. Simonov
Universidad Autónoma de Madrid,
nikita.simonov@uam.es

We study a priori estimates for a class of non-negative local weak solution to the weighted fast diffusion equation $u_t = |x|^\gamma \nabla \cdot (|x|^{-\beta} \nabla u^m)$, with $0 < m < 1$ posed on cylinders of $(0,T) \times \mathbb{R}^N$. The weights $|x|^\gamma$ and $|x|^{-\beta}$, with $\gamma < N$ and $\gamma - 2 < \beta \leq \gamma(N-2)/N$ can be both degenerate and singular and need not to belong to the Mouckenhoupt class $\mathcal{A}_2$, a typical assumption for these kind of problems. This range of parameters is optimal for the validity of a class of Caffarelli-Kohn-Nirenberg inequalities, which play the role of the standard Sobolev inequalities in this more complicated weighted setting.

The weights that we consider are not translation invariant and this causes a number of extra difficulties: for instance, the scaling properties of the equation change when considering the problem around the origin or far from it. We will present quantitative upper and lower estimates for local weak solutions, focussing our attention where a change of geometry appears. Such estimates fairly combine into forms of Harnack inequalities of forward, backward and elliptic type. As a consequence, we obtain Hölder continuity of the solutions.

Perturbed eigenvalue problems

Denisa Stancu-Dumitru

Department of Mathematics and Computer Sciences, University Politehnica of Bucharest, 060042 Bucharest &
Research group of the project PN-III-P4-ID-PCE-2016-0035, "Simion Stoilow" Institute of Mathematics of the Romanian Academy, 010702 Bucharest, Romania

denisa.stancu@yahoo.com

Let $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be a bounded domain with Lipschitz boundary. For each $p \in (1, \infty)$ and $s \in (0, 1)$ we denote by $(-\Delta_p)^s$ the fractional (s, p)-Laplacian operator. We study the existence of nontrivial solutions for a perturbation of the eigenvalue problem $(-\Delta_p)^s u = \lambda |u|^{p-2}u$, in Ω, $u = 0$, in $\mathbb{R}^N \backslash \Omega$, with a fractional (t, q)-Laplacian operator in the left-hand side of the equation, when $t \in (0, 1)$ and $q \in (1, \infty)$ are such that $s - N/p = t - N/q$. We show that nontrivial solutions for the perturbed eigenvalue problem exists if and only if parameter λ is strictly larger than the first eigenvalue of the (s, p)-Laplacian. This is based on joint work with M. Fărcăşeanu and M. Mihăilescu [1].

This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P4-ID-PCE-2016-0035.

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ASYMPTOTIC BEHAVIOUR OF NEURON POPULATION MODELS STRUCTURED BY ELAPSED-TIME

Havva Yoldaş

Universidad de Granada - BCAM (Basque Center for Applied Mathematics)
Bilbao, SPAIN
hyoldas@bcamath.org

We consider two nonlinear partial differential equations which describe the dynamics of elapsed-time structured neuron populations and give some improved results for long time asymptotics. The first model we study is a nonlinear version of the renewal equation, while the second model is a conservative drift-fragmentation equation which adds adaptation and fatigue effects to the neural network model. These problems were introduced in [1] and [2].

We prove that both the problems are well-posed in a measure setting. Both have steady states which may or may not be unique depending on further assumptions. In order to show the exponential convergence to a steady state, we use a technique from the theory of Markov processes called *Doeblin's method*. This method was used in [3] for demonstrating exponential convergence of solutions of the renewal equation to its equilibrium. It is based on the idea of finding a positive quantitative lower bound for solutions to the linear problem. This leads us to prove the spectral gap property in the linear setting. Then by exploiting this property, we prove that both models converge exponentially to a steady state.

This is a joint work with José A. Cañizo.

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On an eigenvalue problem involving the fractional (s, p)-Laplacian

Maria Fărcăşeanu

Department of Mathematics, University of Craiova, 200585 Craiova, Romania

&

Research group of the project PN-III-P4-ID-PCE-2016-0035,

"Simion Stoilow" Institute of Mathematics of the Romanian Academy,

010702 Bucharest, Romania

E-mail address: `farcaseanu.maria@yahoo.com`

The goal of this work is to present an eigenvalue problem involving the fractional (s, p)-Laplacian, which possesses on the one hand a continuous family of eigenvalues and, on the other hand, one more eigenvalue, which is isolated in the set of eigenvalues of the problem. This presentation is partially supported by CNCS-UEFISCDI Grant No. PN-III-P4-ID-PCE-2016-0035.

The Henon equation with a critical exponent under the Neumann boundary condition

Sangdon Jin

We consider the critical Henon equation with the homogeneous Neumann boundary condition.

We are concerned on the existence of a least energy solution and its asymptotic behavior as the parameter approaches from below to a threshold for existence of a least energy solution.

List of participants

1. ELISA AFFILI, *Università degli Studi di Milano*
2. ALESSANDRO AUDRITO, *Universidad Autónoma de Madrid (UAM)*
3. BEGOÑA BARRIOS, *Universidad de La Laguna*
4. JUDITH BERENDSEN, *Westfälische Wilhelms-Universität Münster*
5. CRISTINA BRÄNDLE, *Universidad Carlos III de Madrid*
6. ELEONORA CINTI, *Università degli Studi di Bologna*
7. LUCILLA CORRIAS, *Université d?Evry Val d?Essonne*
8. AZAHARA DE LA TORRE, *Università di Padova*
9. FÉLIX DEL TESO, *Norwegian University of Science and Technology*
10. DANIEL ECEIZABARRENA, *Basque Center for Applied Mathematics - BCAM*
11. JØRGEN ENDAL, *Norwegian University of Science and Technology*
12. MARIA FARCASEANU, *University of Craiova*
13. VERONICA FELLI, *Università di Milano–Bicocca*
14. MARCO FONTELOS, *CSIC-ICMAT Madrid*
15. MARÍA DEL MAR GONZÁLEZ, *Universidad Autónoma de Madrid (UAM)*
16. ANDREEA GRECU, *University of Bucharest and "Simion Stoilow" Institute of Mathematics of the Romanian Academy*
17. KATRIN GRUNERT, *Norwegian University of Science and Technology*
18. SANGDON JIN, *KAIST*
19. MIŁOSZ KRUPSKI, *Uniwersytet Wrocławski*
20. SIMON LARSON, *KTH Royal Institute of Technology*
21. JAVIER C. MARTÍNEZ–PERALES, *Basque Center for Applied Mathematics - BCAM*
22. MARÍA MEDINA, *Pontificia Universidad Católica de Chile*